TWO-PHASE FLOW AND HEAT TRANSFER IN BOUNDARY LAYER OF A THIN BODY OF REVOLUTION

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The effect of transverse curvature of a body and of the amount of injection on flow and heat transfer in a two-phase boundary layer of a thin parabolic body of revolution is investigated.

It is well known that injection and suction have a significant effect on flow and heat transfer. Of particular interest is the problem of injection into a boundary layer of a medium which is different in physical properties from the incoming flow. In particular, this interest derives from the possibility of considerable reduction in the resistance of a body by introduction into the contact region of the boundary layer of materials of low density and viscosity in comparison with the fluid in which the moving body is submerged.

Double-layer flow around plane bodies was studied [1] as well as two-phase heat exchange between slabs [2].

We consider the problem of streamline flow and heat transfer for a slightly heated thin body of revolution in a flow of Newtonian fluid which is parallel to the axis of the body. We assume that gas is discharged normally to the oncoming flow at the surface of the body of revolution. We assume there is a stable interface. For thin bodies of revolution, one can neglect the longitudinal pressure gradient induced by the body. Then the system of equations for an established incompressible two-phase boundary layer in a rectangular coordinate system associated with the contour of the body takes the form

$$u_i \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} = \frac{\nu_i}{r} \frac{\partial}{\partial y}\left(r \frac{\partial u_i}{\partial y}\right), \tag{1}$$

$$\frac{\partial (r u_i)}{\partial x} + \frac{\partial (r v_i)}{\partial y} = 0, \tag{2}$$

$$u_i \frac{\partial T_i}{\partial x} + v_i \frac{\partial T_i}{\partial y} = \frac{\nu_i}{r \Pr_i} \frac{\partial}{\partial y}\left(r \frac{\partial T_i}{\partial y}\right). \tag{3}$$

The boundary conditions at the wall are

$$u_1 = 0,\ v_1 = V_0,\ T_1 = T_0 \ \text{for}\ \ y = 0, \tag{4}$$

we have at infinity

$$u_2 \to U,\ T_2 \to T_\infty \ \text{for}\ \ y \to \infty, \tag{5}$$

and at the interface ($y = \delta$)

$$\begin{gathered} \rho_1 \left(u_1 \frac{\partial \delta}{\partial x} - v_1\right) = \rho_2 \left(u_2 \frac{\partial \delta}{\partial x} - v_2\right) = 0, \\ u_1 = u_2,\ T_1 = T_2, \\ \mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y},\ \ \lambda_1 \frac{\partial T_1}{\partial y} = \lambda_2 \frac{\partial T_2}{\partial y}. \end{gathered} \tag{6}$$

Here and below, $i = 1, 2$. We use the subscript 1 for the internal region and the subscript 2 for the external region.

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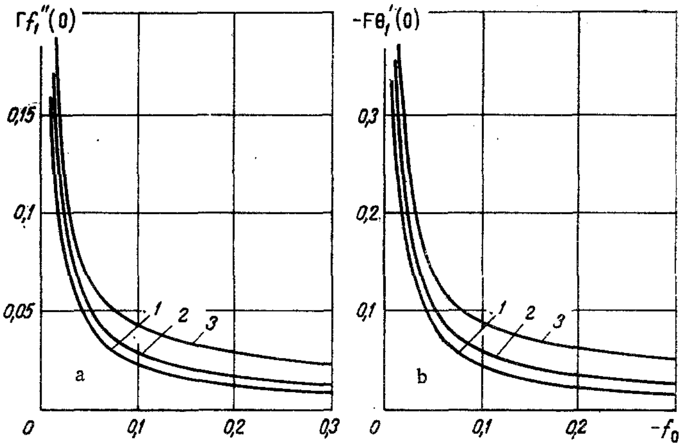

Fig. 1. Surface shear stress (a) and surface heat transfer (b): 1) $A_1 = 0.2$, $A_2 = 0.05$; 2) $A_1 = 2.0$, $A_2 = 0.5$; 3) $A_1 = 8.0$, $A_2 = 2.0$.

We transform the system (1)-(6) by means of new independent and dependent variables

$$\xi = U\int_0^x r_0^2 dx,$$

$$\eta_1 = (2\nu_1\xi)^{-\frac{1}{2}} U \int_0^y r dy, \quad \eta_2 = (2\nu_2\xi)^{-\frac{1}{2}} U \int_\delta^y r dy, \tag{7}$$

$$\Psi_i = (2\nu_i\xi)^{\frac{1}{2}} f_i(\eta_i), \quad \Theta = \frac{T - T_\infty}{T_0 - T_\infty}.$$

After the transformation and calculations similar to those in [3], we find that the conditions providing a self-similar solution will be

$$r_0 = cx^{\frac{1}{2}}, \quad V_0 = c_1 x^{-\frac{1}{2}}, \quad A_i = \frac{2\cos\alpha}{r_0}\left(\frac{\nu_i x}{U}\right)^{\frac{1}{2}}, \tag{8}$$

and the system of equations reduces to the form

$$\left.\begin{aligned} &f_1'''(1 + A_1\eta_1) + A_1 f_1'' + f_1 f_1'' = 0,\\ &\Theta_1''(1 + A_1\eta_1) + A_1\Theta_1' + \mathrm{Pr}_1 f_1 \Theta_1' = 0, \end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned} &f_2'''(1 + A_1\eta_\delta + A_2\eta_2) + A_2 f_2'' + f_2 f_2'' = 0,\\ &\Theta_2''(1 + A_1\eta_\delta + A_2\eta_2) + A_2\Theta_2' + \mathrm{Pr}_2 f_2 \Theta_2' = 0, \end{aligned}\right\} \tag{10}$$

for boundary conditions at the wall

$$f_1 = f_0, \; f_1' = 0, \; \Theta_1 = 1 \;\text{ for }\; \eta_1 = 0, \tag{11}$$

at infinity

$$f_2' \to 1, \; \Theta_2 \to 0 \;\text{ for }\; \eta_2 \to \infty, \tag{12}$$

and at the interface ($\eta_1 = \eta_\delta$; $\eta_2 = 0$)

$$\begin{aligned} &f_1(\eta_\delta) = f_2(0) = 0,\\ &f_1'(\eta_\delta) = f_2'(0), \; \Theta_1(\eta_\delta) = \Theta_2(0),\\ &\Gamma f_1''(\eta_\delta) = f_2''(0), \; F\Theta_1'(\eta_\delta) = \Theta_2'(0). \end{aligned} \tag{13}$$

Here

$$f_0 = -\left(\frac{x}{\nu_1 U}\right)^{\frac{1}{2}} V_0 = \mathrm{const},$$

$$\Gamma = \left(\frac{\mu_1\rho_1}{\mu_2\rho_2}\right)^{\frac{1}{2}}, \quad F = \frac{\lambda_1}{\lambda_2}\left(\frac{\nu_2}{\nu_1}\right)^{\frac{1}{2}}.$$

We note that the curvature parameters A_1 and A_2 are interrelated. Indeed, taking the ratio of A_1 to A_2 we find from Eq. (8) that

$$A_1 = \frac{\rho_2}{\rho_1} \Gamma A_2.$$

A numerical solution of the system (9)-(13) was obtained by the method proposed in [1] for $\Gamma = 0.005$, $F = 0.01$, $Pr_1 = 0.7$, $Pr_2 = 7.0$, $-0.01 \geq f_0 \geq -0.3$.

The exact solutions obtained make it possible to calculate the local coefficient of friction c_f and the local Nusselt number:

$$c_f = \frac{2\tau_w}{\rho_2 U^2} = 2\Gamma \mathrm{Re}_{x_2}^{-\frac{1}{2}} f_1''(0),$$

$$\mathrm{Nu} = \frac{\alpha x}{\lambda_2} = -F \mathrm{Re}_{x_2}^{\frac{1}{2}} \Theta_1'(0).$$

Figure 1 reveals the possibility of considerable reduction in surface shear stress and heat transfer at relatively low rates of gas injection. Figure 1 also indicates that an increase in transverse curvature of the body leads to an increase in surface shear and intensifies heat transfer.

NOTATION

x	is the coordinate calculated along the body generating line;
y	is the coordinate along the normal to the body;
u and v	are the velocity vector components along the axes x and y;
$r = r_0(x) + y\cos\alpha$	is the distance from the body axis to some point at the boundary layer;
$r_0(x)$	is the equation of body contour;
α	is the angle between the tangential line to the body contour and the axis of the body;
T	is the absolute temperature;
ν and μ	are the kinematic and dynamic viscosity coefficients;
λ	is the thermal conductivity;
ρ	is the density;
a	is the thermal diffusivity;
α	is the heat transfer coefficient;
$Pr = \nu/a$	is the Prandtl number;
Ψ	is the stream function;
δ	is the internal boundary layer thickness;
δ_∞	is the conventional thickness of the external boundary layer;
c and c_1	are constants;
L	is the characteristic dimension;
$Re = UL/\nu$	is the Reynolds number.

LITERATURE CITED

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